Letters to the Editor

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APPLICATIONS OF THE NON-DEGENERATE STATISTICAL RELATION TO BINDING ENERGY CHARACTERISTICS IN NUCLEI

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We have derived a statistical relation for the number of occupied states in nuclei (Dutta, 1966, eq. 6) which may be stated as,

$$\begin{aligned} A/2 &= \{k_c A^5 f(A)\} \exp(-E_n{}^0(A)+E_f)/KT \\ &\quad -\{k_c k_f A^5 f(A)\} \exp(-E_n{}^0(A)/KT) \\ &= 2.327\times 10^4 A^5 f(A) \exp(-E_n{}^0(A)/0.40) \qquad \ldots \quad (1) \end{aligned}$$

where $f(A) = 1+4.2 \exp(-3\ 27\times 10^{-2}A^2)$.

The first factor in the first equation above, gives us the number of cells $C(A)$ in different nuclei A slightly modified form of $f(A)$ as $1+4\ 32 \exp(-2.29\times 10^{-2}A^2)$, with $k_c = 3\ 907\times 10^{-3}$ gives us a more rational mode of increase of cell numbers as $C(4) = 16$; $C(8) = 16C(4)$; $C(16) = 16C(8)$; $C(32) = 32.C(16)$ and so on for further duplicated nucleon numbers. It gives us $k_f = 5.955\times 10^6$ and hence $E_f = 6.24$ MeV, when $KT = 0.40$. With the changed expression for $f(A)$, we have also put $E_f = 6\ 226$ MeV and $KT = 0.399$ MeV for better correspondence. E_f becomes the extrapolated limiting energy per nucleon in a single combination particle, on the basis of $E_n{}^0(A)$ values for small nuclei

We now consider the relations (Dutta, 1966, eq 7) for nuclear binding energies. It may be expressed as,

$$E_n{}^0(A) = E_n{}^0(A)[2k_c A^4 f(A) \exp(-E_n{}^0(A)+E_f)/KT] \qquad \ldots \quad (2)$$

The energy of any other form $\epsilon_n{}^0(A)$, associated with the nucleons in a nucleus, is expressed by the modified relation

$$\epsilon_n{}^0(A)/E_n{}^0(A) = [2k_c A^4 f'(A) \exp(-E_n{}^0(A)+E'_f)/(KT)'] \qquad \ldots \quad (2a)$$

The energy under consideration, may be positive or negative and might be related to a single nucleon as in the case of the Λ-particle or distributed throughout the nucleons in a nucleus The relationship suggests that the required ratio is primarily determined by the change in the exponential form with respect to $E_n{}^0(A)$. The change in (KT) and E_f tells us about the comparative rate of variation of the considered energy in relation to $E_n{}^0(A)$ and also about the starting magnitude We may carry this idea to the minimum values of the total deducted energy, $U_n{}^0(A)$, per nucleon in different nuclei or separately to the coulomb and asymmetry energies per nucleon $U_{cn}{}^0(A)$ or $U_{an}{}^0(A)$ The parameters E'_f and $(KT)'$ are obtained by trial with the required values The complete set of relations may be put as follows

$$
\begin{aligned}
E_n{}^0(A) &= E_n{}^0(A)[2k_cA^4f(A)\exp(-E_n{}^0(A)+6\ 226)/0\ 399] \\
&\equiv E_n{}^0(A)[2k_ck_fA^4f(A)\exp(-E_n{}^0(A)/0.399] \\
\Lambda BE &= E_n{}^0(A)[2k_ck_fA^4\exp(-E_n{}^0(A)/0\ 42)] \qquad \ldots \quad \text{(2b)} \\
U_n{}^0(A) &= E_n{}^0(A)[2k_cA^4f(A)\exp(-E_n{}^0(A)+4\ 815)/0\ 4585] \\
U^0{}_{cn}(A) &= E_n{}^0(A)[2k_cA^4f(A)\exp(-E_n{}^0(A)+4.99)/0.443] \\
U^0{}_{an}(A) &= E_n{}^0(A)[2k_cA^4f(A)\exp(-E_n{}^0(A)+0.52/0.643]
\end{aligned}
$$

One may calculate from the set of relations (2b), above, the optimum binding energy E^* of a nucleus, the equivalent uniform radius R, corresponding to the nuclear coulomb energy or the optimum proton number Z_0 in a nucleus. These would be determined by the relations

$$
\begin{aligned}
E^* &= A(E_n{}^0(A)-U_n{}^0(A)). \\
R &= 3/5\ \ Z(Z-1)e^2/(A\cdot U^0{}_{cn}(A). \qquad \text{(3)} \\
A-2Z_0 &= A(U^0{}_{an}/C)^{\frac{1}{2}}
\end{aligned}
$$

where the parameter 'C' is taken as 21.3. The calculated as well as the experimental values of E^* and Z_0 (Konig *et al*, 1962) and of R (Hofstadter, 1956) are shown in the following tables ·

TABLE I

(E*, the binding energies of the most strongly bound nuclei)

A	4	5	8	12	14	19	31	55	111	150	222	250
E*(Calc)	26 6	34 3	58 2	99.3	108	151	259	475	949	1239	1705	1868
E*(expt)	28 3	27.3	56.5	97 1	105	148	263	482	948	1240	1708	1870

TABLE II

(R, the equivalent uniform radius of neuclei)

Nucleus	C^{12}	Mg^{24}	Si^{28}	S^{32}	Ca^{40}	V^{51}	Co^{59}	In^{115}	Sb^{122}	Au^{197}	Pb^{207}	Bi^{209}
R (Calc)	2 90	3 76	4.10	4.35	4 90	4 52	5.02	5.97	5 88	6 84	6.89	7 03
R. Hofstadter	3 04	3 84	3 92	4.12	4.54	4.63	4.94	5 80	5.97	6.87	7 00	7.13

TABLE III

(Z_0, the optimum proton number in a nucleus)

A	16	32	51	59	111	115	150	208	250
Z_0(Calc.)	7 8	15 3	23.8	27.3	48 5	50.1	63	82.1	94 5
Z(expt)	8	16	23	27	48	49	62	82	98

The close agreement between the observed and the calculated values brings out the fundamental nature of $E_n^0(A)$ in determining all forms of nuclear energy values by modified exponential relations.

REFERENCES

Dutta, A K , 1966, *Indian J. Phys* , **40**, 181.
Hofstadter, R 1956, *Rev. Mod Phys*, **20**, 214.
Konig, L. A., Mattauch, T. H , and Wapstra, A. H., 1962, *Nucl. Phy.*, **31**, 18.